MATH 611

Homework 3

1. A classic Monte Carlo problem is estimating the value of π by randomly generating points in a unit square. Consider the following picture:

Suppose each side of the square has unit length, and thus the area of the square is one. Then the area of the circle is π /4. If we generate random uniform variables in the unit square, then the fraction of values that land in the circle will be approximately equal to the area of the circle. Use this procedure in R to estimate the value of π. Run your program 1000 times to obtain 1000 estimates of π. Evaluate the mean and variance of your estimates.

* 1. Use Monte Carlo integration to evaluate the integral:
  2. Use a known probability distribution to analytically evaluate this integral and compare the exact value to the MC estimate.

1. If , show both analytically and by using MC integration that . For the MC simulation, use the value .
2. In the social mobility example discussed in class, show that all 3 states are recurrent.
3. Consider a gene composed by d subunits where d is some positive integer and each subunit is either normal or mutant in form. Consider a cell with a gene composed of m mutant subunits and (d-m) normal subunits. Before the cell divides into two daughter cells, the gene duplicates. The corresponding gene of one of the daughter cells is composed of d units chosen at random from the 2m mutant subunits and the 2(d-m) normal subunits. Suppose we follow a fixed line of descent from a given gene. Let be the number of mutant subunits initially present in the nth descendant gene. Then , n≥0 is a Markov chain. Define the state space and transition probability for and show that d is an absorbing state for this chain.